DIFFERENTIAL GEOMETRY OF SURFACES

1. Determine the intersection of the sphere $x^2+y^2+z^2=4$ and planes

$\alpha: z=1$ — $\left[\sqrt{3}\cos t, \sqrt{3}\sin t, 1\right]$

$\beta: x-y=0$ — $\left[\sqrt{2}\cos t, \sqrt{2}\cos t, 2\sin t\right]$

2. Cut cylinder $x^2+y^2=1$ by a plane $x+y+1=0$. Determine cylinder's normal at intersection point [0,–1, ?].

Rulings $[0,-1,t],[-1,0,t], \vec{n}=(0,t,0)$

3. Estimate the length of parallel N30° between points [20° E, 30° N], [80° E, 30° N].

5780 km

4. Find a vector function that represents the curve of intersection of the cylinders $x^2+y^2=16$ and $x^2+z^2=16$.

$[4\cos t, 4\sin t, \pm 4\sin t]$

5. Find a vector function that represents the curve of intersection of the surfaces

a. $\phi: x^2+y^2+z^2=25, \psi: x^2+y^2=9$ — $[3\cos t, 3\sin t, \pm 4]$

b. $\phi: x^2+y^2+z^2=16; \quad \psi: z=2$ — $\left[\sqrt{12}\cos t, \sqrt{12}\sin t, 2\right]$

c. $\phi: x^2+y^2=9; \quad \psi: y=2$ — $\left[\pm\sqrt{5}, 2, t\right]$

d. $\phi: x^2+y^2=9; \quad \psi: z+y=2$ — $[3\cos t, 3\sin t, 2-3\sin t]$

e. $\phi: x^2+y^2-z^2=1; \quad \psi: z=2$ — $\left[\sqrt{5}\cos t, \sqrt{5}\sin t, 2\right]$

f. $\phi: x^2-y^2+z=0; \quad \psi: z=0$ — $[t, \pm t, 0]$

g. $\phi: x^2+y^2=z; \quad \psi: x^2+y^2=1$ — $[\cos t, \sin t, 1]$

6. Cut the paraboloid $z=x^2+y^2$ by the planes x = –3 and y =–3. Determine the tangent lines to the curves of intersections at common point T = (–3, –3, ?).

7. Find the general equation of tangent plane to the cylinder $x^2+z^2=1$ at a point T = [1,0,0].

$x=1$

8. Find the tangent plane τ to the hyperbolic paraboloid ϕ at the point T and cut the paraboloid by its tangent plane, i.e. determine the curve of intersection $\phi \cap \tau$.

a. $\phi: x^2-y^2+z=0; T[0,0,0]$ — $z=0, [t, \pm t, 0]$

b. $\phi: x^2-y^2+z=0; T[2,1,-3]$ — $4x-2y+z-3=0, [t, t-1, 1-2t], [t, 3-t, 9-6t]$

9. Cut hyperbolic paraboloid z = xy by planes $x=\frac{1}{2}$ and $y=\frac{1}{2}$. Determine the tangent plane and normal of the surface at the point $T=\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{4}\right)$.

10. Determine the tangent plane to the paraboloid of revolution $z=\frac{x^2+y^2}{9}$ at point T = (–3, 3, ?). Construct the normal and find its intersection with axes of revolution.

(0, 0, 6.5)

11. Determine the tangent plane to the hyperbolic paraboloid $z = xy$ at points O = (0,0,0) and T = (2, 1, 2)

z = 0, x + 2y – z = 2

12. At what point on the paraboloid $z = x^2 + y^2$ is the tangent plane parallel to the plane x + 3y + 2z = 1?

$\left(\frac{-1}{4}, \frac{3}{4}, \frac{10}{16}\right)$

13. Are there any points on the hyperboloid $x^2 - y^2 - z^2 = 1$ where the tangent plane is parallel to the plane $z = x + y$?

no

14. Show that tangent plane to the cone $z^2 = x^2 + y^2$ at point T = (1,1,?) passes through the origin.

$x + y - \sqrt{2}y = 0$

15. Find minimum of the function $z = x^2 + y^2 - 2x - y$ (1, 3, 4)

Find local extremes of the function z = f(x,y) under condition (1,0.5)

a. $x^2 + y^2 = 4$ 8.47

b. x + y = 2 –1.125

16. Determine level curve of hyperbolic paraboloid $z = \frac{x^2 - y^2}{9}$ at the point A = (–2,–1, ?). Find the direction of the greatest slope (fall line).

$grad = \left(\frac{-4}{9}, \frac{2}{9}\right)$

17. Sketch the contour map (level curves) of the plane given by three points A,B,C on the coordinate axes; A = (2, 0, 0), B = (0, 3, 0) and C = (0, 0, 3).

$grad = \left(\frac{-3}{2}, -1\right)$

18. Sketch the contour map (level curves) and gradient field of the

a. right cone $x^2 + y^2 = 2z^2$ $\left[x^2 + y^2 = 2z_0^2\right]$

b. circular paraboloid $z = x^2 + y^2$

c. hyperbolic paraboloid $z = \frac{xy}{2}$

19. Find the directional derivative of the function z = f(x,y) at the given point A in the direction of the vector *v*.

a. $z = \frac{x^2}{2} + \frac{y^2}{2}$; A = (1, 3), *v* = (1,0) $grad = (1, 3)$, $z'_v = 1$

b. $z = \sqrt{x^2 + y^2}$; A = (2, 2) *v* = (1,0) $grad = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right), \frac{\partial z}{\partial x} = \frac{\sqrt{2}}{2}$

c. $z = \frac{xy}{2}$ A = (-2, -2) *v* = (–1,1) *slope* = 0.

20. Find the curve on the surface $z = 4 - \sqrt{x^2 + y^2}$ passing through the point A with given slope. Determine direction of vertical cutting plane.

a. A = (2, –1, ?), slope = –1 *v* = (2, –1)

b. A = (2, –1, ?), slope = 0.44 *v* = (0, 1)

c. A = (0, –2, ?), slope = 0 *v* = (1.0)