DIFFERENTIAL GEOMETRY OF CURVES I

1. Draw the plane curve $X(t)=\left[t^2,t^3\right]$ and write its implicit equation. Determine the tangent and normal of curve at points X(t = 1).

$x^3-y^2=0, t=[1+2t;1+3t], n=[1-3t,1+2t]$

2. Find the intersection points of curve $X(t)=\left[t^2,t-t^2\right]$ and coordinate axes *x* and *y*. Find all points on the curve with horizontal tangent lines.

$[0,0],[1,0],\left[\frac{1}{4},\frac{1}{4}\right]$

3. Verify the rational parametrization of the circle, find center and radius.

$X(t)=\left[\frac{1-t^2}{1+t^2},\frac{2t}{1+t^2}\right]$

S[0, 0] *r* =1

4. Prove that the implicit formula $\sqrt[3]{x^2}+\sqrt[3]{y^2}=\sqrt[3]{a^2}$ and parametric vector equation.

$X(t)=\left[a\cos^3 t,a\sin^3 t\right]$ represent the same planar curve. Find singular points, i.e. points with zero derivative vector.

[*a*,0], [0, *a*], [-*a*, 0], [0, -*a*]

5. Determine all points on the curve with horizontal and vertical tangent lines.

a. $X(t)=\left[a\cos t,b\sin t\right]$ — [*a*,0], [0, *b*], [-*a*, 0], [0, -*b*]

b. $X(t)=\left[a\sqrt{1-t},\sqrt{t}\cdot b\right]$ — nothing, these points has no derivative

6. Describe a curve $X(t)=\left[\operatorname{cotg} t,\sin^2 t\right]$ explicitly, as a graph of a function y = f(x) .

$y=\frac{1}{x^2+1}$

7. Draw the curve with polar equation $\rho=2\cos\phi$. Determine the tangent line to the curve at a point $\rho=0$.

Circle, tangent: x =2

8. Draw the curve with polar equation $\rho=\phi$. Determine the tangent line to the curve at point T = [0, 0].

t: *y* =0

9. Determine the value of parameter *c* so that the cycloid $X(t)=\left[t-c\sin t,1-c\cos t\right]$ has singular points (cusps).

c=1, singulární body $\left\{[0,k\cdot 2\pi];k\in Z\right\}$

10. Calculate the arc length along the cycloid $X(t)=\left[t-\sin t,1-\cos t\right], t\in\langle 0,2\pi\rangle$ corresponding to one complete revolution of a circle.

l=8

11. Determine the instantaneous velocity and acceleration of cycloidal motion $X(t)=\left[t-\sin t,1-\cos t\right], t\in\langle 0,2\pi\rangle$ at time t = 2s.

v = (1,42; 0,91), *a* = (0,91; -0,42)

12. Determine the length of a part of asteroid $X(t)=\left[\cos^3 t,\sin^3 t\right]$ between two singular points.

$l(t)=\frac{3}{2}\sin^2 t; l\left(\frac{\pi}{2}\right)=\frac{3}{2}$

DIFFERENTIAL GEOMETRY OF CURVES II

13. Determine the arc length of the curve $X(t)=\left[t,\sqrt{t^3}\right],\ t\in\langle 0,1\rangle$.

$$\frac{13\sqrt{13}-8}{27}$$

14. Determine the points on a curve $X(t)=\left[3t^2,2t\right]$ with minimal and maximal value of curvature. Draw osculating circle for all these points.

$[0,0];\ k(0)=\frac{3}{2}$

15. Find the point on curve $X(t)=[t,\tan t]$ with the greatest value of curvature. Estimate maximal curvature with 3 decimal places precision.

maximum: k(0,6587) = 0,369

16. Write down the curvature function for graph of the function $y=e^x$. Estimate maximal curvature with 3 decimal places precision.

$k(x)=\frac{e^x}{\sqrt{\left(e^{2x}+1\right)^3}}$, maximum k=0,385

17. Draw the evolute (locus of centers of curvature) of the cycloid $X(t)=[t-\sin t,1-\cos t],t\in\langle 0,2\pi\rangle$.

cycloid

18. Determine the radius of curvature of Archimedean spiral $\rho=\phi$ at point $\phi=\pi$.

r = 3,02

19. Determine the length of Euler spiral $k=\frac{l}{200}$ for transition between straight road and circular arc of radius r =20 *m*.

l=10,

20. Find the parameter A for Euler spiral $k=\frac{l}{A^2}$, used as a 72m long transition curve between straight road and circular arc r=*200 m*.

A=120

21. Draw the Euler spiral $k=\frac{l}{50^2}$ and its cubic taylor approximation.

$$y=\frac{x^3}{6.50^2}$$

22. Use the cubic function for smooth connection AB (smooth first derivative) between two horizontal parts. $y=0$ and $y=1$. A = (0,0) a B = (1,1).

$y=-2x^3+3x^2$

23. Determine the cubic approximation of $y=\tan x$ on interval $\left(-\frac{\pi}{6},\frac{\pi}{6}\right)$. Determine the maximal error of approximation.

e.g. $y=x+\frac{x^3}{3}$, error 0,006

24. Draw the quadratic Taylor polynomial of function $y=\sin x$ at point x = 0. Determine the maximal error on interval (0,15°).

$y=x$, error 0,003