Linear transformations

All linear transformations fix the origin O(0,0). Equations can by written by 2x2 matrix $X' = AX;\ A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

The matrix Rot={{cos(α), -sin(α)}, {sin(α), cos(α)}} acts as a rotation of E^2 about the origin.
Use the command *ApplyMatrix(M, c)* to get image of object *c* under transformation defined by *M*.

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1. Map the parabola $x^2 - 4y + 4 = 0$ in linear transformation $X' = AX;\ A = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$. Determine the intersection of parabola and its image.

G = (0,838; 1.176)

2. Write the matrix representation of reflection in the straight line y = 2x

3. Determine all fixed point of linear mapping $X' = AX;\ A = \begin{pmatrix} 0.5 & 0.87 \\ 0.87 & -0.5 \end{pmatrix}$

[approx. *y* = tg(π/6) x]

4. Estimate parameter *a* so that the matrix $A = \begin{pmatrix} a & -0.707 \\ 0.707 & a \end{pmatrix}$ represents revolution with angle π/4.

a = 0,707

5. Estimate parameter *a* so that the matrix $A = \begin{pmatrix} -0{,}5 & a \\ a & 0{,}5 \end{pmatrix}$ represents reflection in line.

a = 0,866

6. Find the matrix for 90° clockwise rotation in the plane around the origin. Draw the unit circle and its image.

7. Determine the parameter *a* so that the image of parabola $y = x^2$ pass through a point B = (2, 2)

Assume linear transformation $X' = AX;\ A = \begin{pmatrix} 0 & a \\ 1 & 0 \end{pmatrix}$.

a = 0,5

8. Determine the parameter *a* so that the ellipse $4x^2 - 4xy + 5y^2 = 16$ transform to the circle.

Assume linear transformation $X' = AX;\ A = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}$.

a = – 0,5

9. Describe the composition of reflection in line o_1: x = 0 followed by reflection in line o_2: x = y. Draw the unit circle and its image. Find the reverse transformation.

Affine transformations

When the origin moves to the point O' = (m, n), matrix representation has a form $A = \begin{pmatrix} a & b & m \\ c & d & n \\ 0 & 0 & 1 \end{pmatrix}$

The matrix Rot={{cos(α), -sin(α),0}, {sin(α), cos(α),0},{0,0,1}} acts as a rotation of E^2 about the origin.
Use the command *ApplyMatrix(M, c)* to get image of object *c* under transformation defined by *M*.

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1. Assume affine transformation $X' = AX;\ \ A = \begin{pmatrix} 0 & -1 & a \\ 1 & 0 & -1 \\ 0 & 0 & 1 \end{pmatrix}$.

a) Map conic $c: x^2 + y^2 - 7x - 3y + 12 = 0$ for $a = 6$. Determine the parameter a so that the image of ellipse $c: 2x^2 + y^2 - 4x - 3y = -2$ has axis parallel with coordinate lines. Find all fixed points.

$a = 1.5$, FP = (3.5, 2.5)

2. Find the matrix for reflection across a line y = x + 2. Draw the unit square and its image.

O' = (–2, 2)

3. Write the matrix representation of reflection in the straight line y = 2x+1

4. Write the matrix representation of the plane rotation around S = (1,2) with a angle α =45°.

O' = (1.71,–0.12)

5. Describe composition of rotation about origin by 60° and translation by a vector v = (1,2). Determine all fixed point of the resultant transformation.

Rotation, FP = (–1.23, 1.87)

6. Describe composition of translation by a vector v = (1,2) and rotation about origin by 60°. Determine all fixed point of the resultant transformation.

Rotation, FP = (–2.23, –0.13)

7. Describe the composition of two reflection in parallel lines o_1: x = 0 and o_2: x =2. Draw the image of a point A = (1, 3).

Translation u = (4,0)

8. The matrix $A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & -2 \\ 0 & 0 & 1 \end{pmatrix}$ acts as a reflection of E2. Find the axis of reflection.

x – y = 2

9. The matrix $A = \begin{pmatrix} 0 & -1 & 4 \\ 1 & 0 & -1 \\ 0 & 0 & 1 \end{pmatrix}$ acts as a rotation of E2. Find the center and the angle of rotation.

FP = (2.5; 1.5), α = 90°